A Statistical Model Analysis of (p,α)-Reactions on ²⁶Mg, ³⁷Cl and ⁴⁵Sc

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Dedicated to Professor Dr. W. Gentner on the occasion of his 60th birthday

The reactions $^{26}{\rm Mg}(p,\alpha)^{23}{\rm Na}$, $^{37}{\rm Cl}(p,\alpha)^{34}{\rm S}$, and $^{45}{\rm Sc}(p,\alpha)^{42}{\rm Ca}$ have been studied. Excitation functions and angular distributions were measured for proton energies 9 to 13.26 MeV, 11 to 11.952 MeV, and 8.5 to 9.412 MeV in steps of 20 keV, 8 keV, and 6 keV, and the transitions into 4, 3, and 5 final states have been resolved, respectively. All the excitation functions exhibit strong fluctuations and were analysed in terms of the various correlation functions, from which the mean level width Γ , the normalised variance, and the amount of direct interaction contribution was derived. Statistical model calculations have been used to fit the averaged angular distributions and an attempt was made to estimate the J-dependence of the mean level width Γ in the compound nuclei excited at about 20 MeV.

During the last few years nuclear reaction studies on medium weight nuclei have demonstrated clearly the existence of nuclear cross section fluctuations. These fluctuations were first predicted by Ericson 1 in 1960 and since this time many contributions both theoretical 2 and experimental 3 have been made.

In the experiments described in this paper an attempt was made to evaluate the average width Γ of the compound nucleus levels and the ratio of Γ_0/D_0 even in the region where the levels are overlapping as well as the magnitude of fluctuations by means of correlation functions. Besides that, we also tried to estimate the amount of direct interaction contribution to the cross section. In order to obtain

the informations about all these quantities over a fairly broad range of nuclei, the reactions $^{26} Mg\,(p,\alpha)\,^{23} Na,~^{37} Cl\,(p,\alpha)\,^{34} S~$ and $^{45} Sc\,(p,\alpha)\,^{42} Ca$ have been studied extensively. Although accounts of the first two reactions have already been published 4,5 and the data of the third one are going to be combined with data taken at Argonne 6, it is worthwhile to summarise some results of all three in this work. The advanced analysis of the first two reactions presented here led to a deeper understanding of the results given earlier.

The three experiments were chosen for the following principal reasons:

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⁴ B. W. Allardyce, P. J. Dallimore, I. Hall, N. W. Tanner, A. RICHTER, P. VON BRENTANO, and T. MAYER-KUCKUK, Nucl. Phys., in press.

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J. R. Huizenga, A. Katsanos, A. Bamberger, and T. Mayer-Kuckuk, to be published.



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1. There are several previous (p, α) -measurements on medium weight nuclei which have shown that the direct interaction contribution to the cross section is relatively small and therefore the statistical theory is applicable without too much complications.

2. With the proton beam from the Tandem Van DE Graaff accelerator it is possible to excite the nucleus in an energy region where the levels overlap, that means normally several MeV above the neutron emission threshold. Furthermore, it is anticipated from the small energy spread of the beam that the experimentally observed structures in the excitation functions could be resolved completely.

3. The fact that both the α -particles and either the target nucleus or the final nucleus have zero spin leads to a simple angular momentum decomposition of the scattering amplitudes. In this case the strength of the fluctuations should be large.

4. The α-particle transitions to the ground state and to the first few excited states in the final nuclei ²³Na, ³⁴S, and ⁴²Ca could be measured easily with solid state counters.

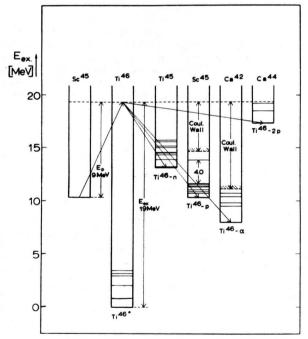


Fig. 1. The decay scheme of the compound nucleus ⁴⁶Ti.

⁸ P. A. Moldauer, Phys. Letters 8, 70 [1964].

As an example, Fig. 1 shows the decay scheme of the compound nucleus 46 Ti. In the compound system the open exit channels are mainly the neutron, proton and α -particle channels. For exact numerical calculations of the transition probabilities into the various final states the knowledge of the spins and parities of the levels is required up to a certain excitation energy. These quantities are not known well enough, but it has been shown explicitly that the sum over all exit probabilities can be replaced by the following expression $^{7, \, 8}$

$$\sum_{c^{\prime\prime}} T_c^{\prime\prime} = \frac{2 \pi}{D_J} \langle \Gamma_{J\pi} \rangle$$

where D_J is the mean spacing of levels with spin J and $\langle \Gamma_{J\pi} \rangle$ is the mean level width in the compound nucleus depending on the angular momentum J and parity π . Starting from a work by Vonach and Huizenga⁹, von Brentano, Eberhard, and Stephen¹⁰ recently have derived the following formula for the J-dependence of Γ

$$\Gamma_J = \Gamma_0 \exp\left\{-J(J+1)/s^2\right\} \tag{1}$$

where

$$\frac{1}{s^2} = \frac{1}{2 \, \sigma^2_{\text{res}} + 2 \, m \, R^2 \, T} - \frac{1}{2 \, \sigma_c^2}$$

Here Γ_0 is the mean level width for the lowest J-value to be formed through angular momentum coupling in the compound nucleus, $\sigma_{\rm res}^2$ and $\sigma_{\rm c}^2$ are the spin cut-off parameters of the residual nucleus and of the compound nucleus, respectively, m is the mass of the neutron, R the radius of the residual nucleus and T its nuclear temperature. The term $2\ m\ R^2\ T$ is a small correction and has been neglected in our cases. The assumptions from which the formula for the J-dependence of the mean level width T has been derived are stated explicitly in ref. 9 and 10 . In addition, Eq. (1) holds only if the condition $J \leq 2\ \sigma_{\rm res}^2$ is fulfilled.

Formula (1) has a consequence on the calculation of the averaged differential cross section. The Hauser-Feshbach expression given in previous publications ^{5, 11} is slightly altered now and has to be replaced by the following relation:

⁷ J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, J. Wiley, New York 1952, p. 386.

⁹ H. K. Vonach and J. R. Huizenga, Phys. Rev. 138, B 1372 [1965].

¹⁰ P. von Brentano, K. Eberhard, and R. O. Stephen, private communication.

A. RICHTER, W. VON WITSCH, P. VON BRENTANO, O. HÄUSSER, and T. MAYER-KUCKUK, Phys. Letters 14, 121 [1965].

$$\left\langle \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\Theta) \right\rangle = \frac{D_0}{\Gamma_0} \frac{\lambda^2}{8 \pi (2 I + 1) (2 i + 1)} \sum_{\substack{cc' \\ LJ}} A_{cc'}^{LJ}(\Theta)$$
(2)
$$\frac{T_c T_{c'}}{(2 J + 1) \exp\{-J(J + 1)/2 \sigma^2_{\mathrm{res}}\}}$$

Here the spins of the incoming particle and of the target nucleus are denoted by i and I, $\overline{\lambda}$ is the wave length of the bombarding particles and $c=(\alpha,l,s)$ stands for the initial state in the reaction. The primed quantities refer to the final state. The factor A contains the geometrical properties of the differential cross section, the T's are optical model transmission coefficients.

With the help of this formula, one can estimate the spin cut-off parameter of the residual nuclei and the ratio of Γ_0/D_0 by fitting the averaged angular distribution. Using all these quantities, the mean width Γ deduced from the excitation functions can be related to the value of Γ_J/Γ_0 from Eq. (1). The knowledge of the Hauser-Feshbach cross section is also the supposition for the calculation of the various correlation coefficients from which the number of effective channels is obtained.

Apparatus

Target Chamber and Detectors

The proton beam from the Heidelberg Tandem Van DE Graaff accelerator was used throughout these three experiments. The analysed beam passed through a pair of slits before it was focussed by a quadrupole

lens onto the target in the center of the scattering chamber. This chamber shown schematically in fig. 2 was a cylindrical cup of 50 cm diameter. Opposite to the entrance of the beam into the target chamber, a FARADAY cup was mounted to collect the total charge. Secondary electrons were prevented from escaping the FARADAY cup by means of a strong permanent magnet positioned close to the mouth of the cup and by a suppressor ring kept at a potential of $-400 \,\mathrm{V}$. In fig. 2 several detectors are indicated which were fixed at the turnable lid of the target chamber and could be moved into any angular position required. The detectors were silicon surface barrier counters with a specific resistance between 500 and 6000 Ω cm. The counters were biased just as much as necessary to stop the a-particles in order to discriminate against protons. The pulses from each detector were amplified and then collected in several multichannel analysers. Up to 8 counters were used simultaneously to measure an angular distribution.

Targets

For the 26 Mg (p, α) 23 Na reaction the target was made of 26 Mg, enriched to 99.4%, and evaporated on a carbon foil of about 10 μ g/cm². The target was supplied by AERE Harwell, its thickness was about 100 μ g/cm², estimated by weighing and by measuring the energy loss of 14 N-ions using a magnetic spectrograph.

The 37 Cl targets were made of enriched NaCl (88% 37 Cl) which was evaporated on a carbon backing of $10 \,\mu\text{g/cm}^2$. The target thickness was determined by weighing before and after the evaporation and was about 75 $\mu\text{g/cm}^2$. It was found in the experiment when the target was frequently compared with reference targets that even at beam currents of 600 nA no evaporation of NaCl could be noticed.

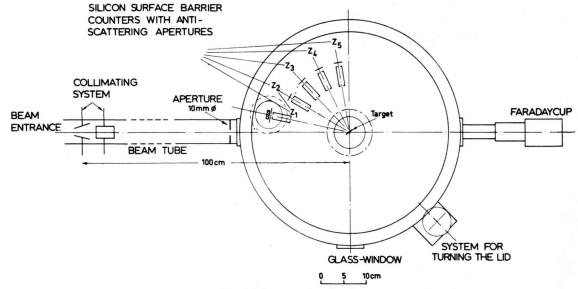


Fig. 2. The scattering chamber.

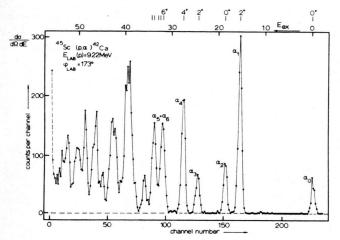


Fig. 3. Pulse height spectrum from the reaction 45 Sc (p,a) 42 Ca. Identified peaks corresponding to the ground state and to the first six excited states transitions are seen. At the low energy side of the spectrum the outset of proton counts is seen.

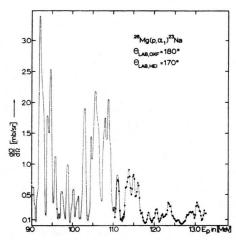


Fig. 4. Excitation function for the 26 Mg $(p,\alpha_1)^{23}$ Na reaction. The full line corresponds to the measurement done at Oxford, the broken line to that at Heidelberg. The laboratory angles indicated on the figure should read in fact $\Theta_{\rm lab}$, $o_{\rm xf.}=177^\circ$ and $\Theta_{\rm lab}$, $H_{\rm ei}=175^\circ$.

The scandium 45 isotope has also been evaporated on a carbon backing. The two targets used in the measurements have been 71 ± 7 and $32\pm3~\mu\mathrm{g/cm^2}$ thick, respectively. These values have been determined by weighing and by RUTHERFORD scattering of 6 MeV α -particles at 40° and 60° . The COULOMB barrier for α -particles is about 8.5 MeV for $^{45}\mathrm{Sc}$, so under forward angles one can be sure to detect α -particles scattered in the COULOMB field only. It was found through elastic scattering of protons under backward angles that the target was contaminated by the elements $^{14}\mathrm{N}$, $^{16}\mathrm{O}$, $^{19}\mathrm{F}$ and $^{28}\mathrm{Si}$.

Beam Spread

In the Ericson type of experiments the energy resolution in the entrance channel of the reaction has to be less than the observed width of the excitation

function. The energy spread of the beam was investigated by the measurement of the width of the 7.74 MeV resonance in 29P, using the 28Si(p,p1) reaction. The energy of the bombarding protons was about 5.2 MeV. Allowing for the target thickness contributing to the measured width of this resonance, which is stated to be ≥ 2 keV 12, the total energy spread of the beam incident on the target was about 3 keV. From observation of structures less than 4 keV wide in the ⁵⁶Fe(p,p) reaction at 10 MeV 13, however, one can be almost sure that the beam spread was not worse here than the one found at 5 MeV. The energy loss of protons in the targets ranged from about 2 to 4 keV. So, for an energy loss of 3 keV, the energy resolution of the protons interacting with the nuclei in the target was better than 5 keV.

The general experimental facilities in the three experiments are shown in Table 1.

	$E_{ m p}$ (MeV)	$\begin{array}{c} \mathbf{Energy} \\ \mathbf{steps} \\ (\mathbf{keV}) \end{array}$	Energy resolution (keV	$\Delta\Theta$ (degree)	$\Delta\Omega$ (10 ⁻³ sr)	$I dt$ (μCb)	Target $(\mu g/cm^2)$
²⁶ Mg(p, α) ²³ Na	9-13.26+	20	< 5	2.5	2.3	150	100
$^{37}{\rm Cl}({\rm p},\alpha)^{34}{\rm S}$	11 - 11.952	8	<5	2.5	2.3	200	75
$^{45}\mathrm{Sc}(\mathrm{p},\alpha)^{42}\mathrm{Ca}$	8.5-9.412	6	< 5	3.5	3.0	200	$\begin{array}{c} 71 \\ 32 \end{array}$

⁺ The energy range measured in Oxford 4 is included.

Table 1. The main experimental features of the three (p, α) -reactions.

¹² M. W. Brenner, A. M. Hoogenboom, and E. Kashy, Phys. Rev. 127, 947 [1962].

¹³ T. MAYER-KUCKUK, J. ERNST, W. VON WITSCH, and P. VON BRENTANO, Compt. Rend. Congr. Intern. Phys. Nucl., Ed. Centre National de la Recherche Scientifique, Paris 1964, Vol. II, p. 731.

Experimental Results

As an example for the three (p, α) -reactions regarded here, Fig. 3 displays a representative α -particle spectrum from the reaction ${}^{45}Sc(p, \alpha) {}^{42}Ca$. The spectrum clearly shows identified peaks corresponding to the ground state and to the first six excited state transitions. The proton pulses indicated in the figure have been cut off by the biased post amplifier. The spectra were recorded in multichannel analysers simultaneously at center of mass angles ranging from 175° to 33°. In the 45Sc-case the raw data from the kicksorter were automatically punched out onto paper tapes and analysed with a GIERprogram 13a. In all cases no underground substraction has been found to be necessary. In the Figs. 4-9out of the large supply of reliable data some excitation functions are seen.

Figs. 4 and 5 show the excitation functions for the $^{26}{\rm Mg}\,({\rm p},\alpha)\,^{23}{\rm Na}\,$ reaction to the first and second excited states of $^{23}{\rm Na}\,$ under laboratory angles of $175\,^{\circ}$ and $130\,^{\circ}$. The Oxford and Heidelberg data taken at the same angles were normalised to each

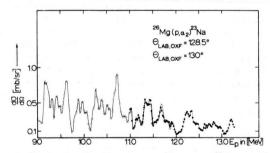


Fig. 5. Excitation function for the 26 Mg $(p,\alpha_2)^{23}$ Na reaction. The Oxford and Heidelberg data overlap in the region from 11 to 12 MeV.

other in the overlap region. As can be seen from the pictures, the shape of the curves measured in the two laboratories coincide almost exactly. The data have been taken in 20 keV steps using the information about the mean level width Γ already known from the $^{26}{\rm Mg}(p,p)$ — experiment 14 at the same excitation energy of the compound nucleus $^{27}{\rm Al}$. All the observed structures have been reproduced and a measurement in 10 keV steps was made to proof that there are no further fine structures present.

The Figs. 6 and 7 are concerned with the reaction $^{37}\text{Cl}(p,\alpha)^{34}\text{S}$. Excitation functions are displayed for the first and second excited state transitions. The data are restricted to an energy range from 11 to

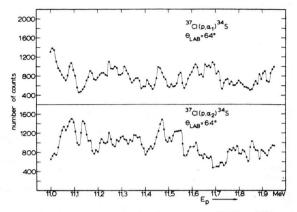


Fig. 6. Excitation functions for the reactions ${}^{37}\text{Cl}(p,\alpha_1){}^{34}\text{S}$ and ${}^{37}\text{Cl}(p,\alpha_2){}^{34}\text{S}$ at the same laboratory angle $\Theta_{\text{lab}}=64^{\circ}$.

11.92 MeV. The excitation function was measured in 8 keV steps. The reproducibility of the data and the energy resolution in the entrance channel of the reaction already stated support that the structures in the excitation functions are clearly resolved.

Four examples of excitation curves from the reaction $^{45}\mathrm{Sc}(\mathrm{p},\alpha)^{42}\mathrm{Ca}$ are seen in the Figs. 8 and 9. Fig. 8 presents three excitation functions for α_0 , α_1 , and α_2 at the same angle $\Theta_{\mathrm{lab}}=173^{\circ}$. The cross sections were measured in 6 keV steps and are reproduced partially in 4 keV steps as it is indicated in the tail of the curves. The excitation function for the α_1 -transition at $\Theta_{\mathrm{lab}}=137^{\circ}$ finally emphasises one main characteristic of the differential cross section measured in the three reactions reported here: the decreasing average with increasing bombarding energy of the protons, mainly due to the fact that more channels open up.

Another significant aspect illustrated in the Figs. 4-9 is the strong variation of the differential cross section as a function of energy. The oscillatory behaviour of the cross section is most pronounced at backward angles. The cross sections in the $^{45}{\rm Sc}(p,\alpha)^{42}{\rm Ca}$ reaction are smaller by a factor of about 10 than those obtained in the $^{26}{\rm Mg}(p,\alpha)^{23}{\rm Na}$ reaction, probably due to the higher level density in

¹³a The program was written by M. Dost.

¹⁴ O. HÄUSSER, A. RICHTER, P. VON BRENTANO, and T. MAYER-KUCKUK, Compt. Rend. Congr. Intern. Phys. Nucl., Ed. Centre National de la Recherche Scientifique, Paris 1964, Vol. II, p. 728.

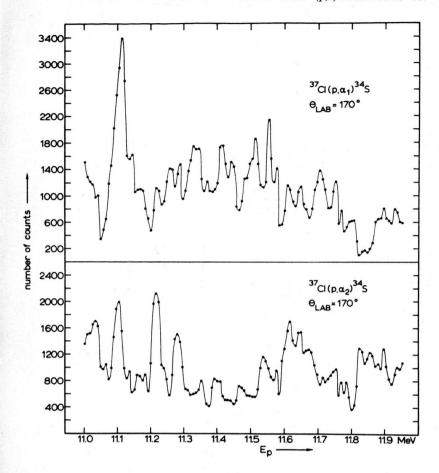


Fig. 7. Same as in Fig. 6 but at $\Theta_{\rm lab} = 170^{\circ}$. From the number of counts on the ordinate the statistical error may be seen.

the compound nucleus and to the penetration of the α-particles through the COULOMB barrier.

The absolute cross section was determined in all three experiments and is indicated in most of the figures displaying cross sections. The principal sources of error are the uncertainty in (i) the target thickness, (ii) the total integrated beam current, (iii) the solid angle of the counters and (iiii) the counting statistics. The item (i) contributes most to the error and is about 10%. So, we estimate that the overall uncertainty in the absolute cross section is less than 15%. The relative value of the differential cross section, however, which in a great deal is sufficient for a fluctuation analysis, is known much better in all three reactions.

More complete sets of experimental data for the $^{26}\text{Mg}(p,\alpha)^{23}\text{Na}$ reaction and for the $^{37}\text{Cl}(p,\alpha)^{34}\text{S}$ reaction are presented in the papers by Allardyce et al. 4 and von Witsch et al. 5 , respectively.

Discussion

The statistical model provides a convenient framework for the understanding of the characteristics of the three (p, a)-reactions reported in the previous section. The fact that the compound nuclei ²⁷Al, ³⁸Ar, and ⁴⁶Ti are excited 7, 10, and 6 MeV, respectively, above the neutron emission threshold gives reason to believe that the statistical continuum condition holds, which is also confirmed by HAUSER-FESHBACH type of calculations. The results of such calculations are seen in the Figs. 10, 11 and 12, where the angular distributions for the reactions 26 Mg (p, α) 23 Na, 37 Cl (p, α) 34 S, and 45 Sc (p, α) 42 Ca are displayed. Assuming a pure statistical reaction, Eq. (2) was used to fit the experimental points. The fitting procedure has already been described in detail 4 and it is seen from the figures that the Hauser-Feshbach curves resemble the experimental cross sections well. The shape of the angular

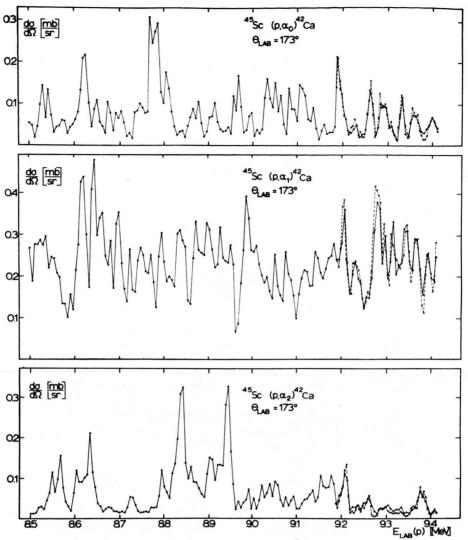


Fig. 8. Excitation functions for the reactions $^{45}\text{Sc}(p,\alpha_0)$ ^{42}Ca , $^{45}\text{Sc}(p,\alpha_1)$ and $^{45}\text{Sc}(p,\alpha_2)$ at $\Theta_{\text{lab}} = 173^{\circ}$ measured in 6 keV steps. On the high energy end of the curves the broken line demonstrates the reproducibility of the data.

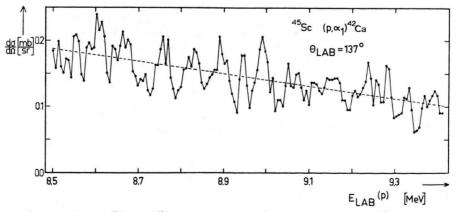


Fig. 9. Excitation function for the ${}^{45}\mathrm{Sc}\,(p,\alpha_1){}^{42}\mathrm{Ca}$ reaction to show the decreasing average with increasing proton energy.

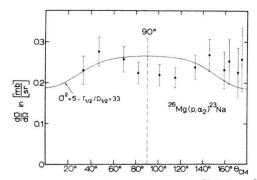


Fig. 10. Andular distribution for the reaction $^{26}\text{Mg}\,(p,\alpha_2)$ ^{23}Na averaged over about 2 MeV proton energy. The error bars on the measured points are f. r. d.-errors. The Hauser-Feshbach fit with the parameters given in the figure is indicated.

distribution depends on the spin cut-off parameter of the residual nuclei. Owing to the fact that the decay of the compound nucleus by nucleons gives the main contribution to the width ^{14a}, $\sigma_{\rm res}^2$ is an average of the two spin cut-off parameters of the residual nuclei reached by proton and neutron emission.

The averaged angular distribution from the reaction $^{26}\text{Mg}(p, \alpha_2)^{23}\text{Na}$ is illustrated in Fig. 10. Like the other observed transitions to the ground, to the first, and to the third excited states this angular

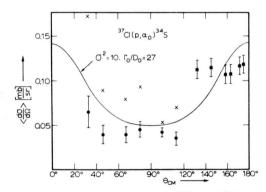


Fig. 11. Angular distribution for the \$\frac{37}{\text{Cl}}(p,\alpha_0)\frac{34}{\text{S}}\$ reaction averaged over the whole measured interval. Crosses represent the uncorrected cross sections, the full dots with f. r. d.-errors show the results after the direct interaction contribution to the cross section has been subtracted. The curve is calculated according to Hauser and Feshbach.

distribution is symmetric about 90° . The second excited state in $^{23}\mathrm{Na}$ has a spin $7/2^{+}$ and the slight anisotropy of the measured cross sections is repro-

¹⁵ I. Hall, Phys. Letters 10, 199 [1964]. — P. J. Dallimore and I. Hall, Nucl. Phys., to be published.

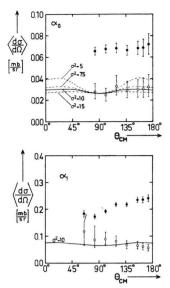


Fig. 12. Averaged angular distributions for the $^{45}\mathrm{Sc}(p,\alpha_0)^{42}\mathrm{Ca}$ and $^{45}\mathrm{Sc}(p,\alpha_1)$ reactions. The open circles represent the statistical cross sections obtained by subtracting the direct reaction distribution. The Hauser-Feshbach fit with the different spin cut-off values given in the figure is indicated, but $\sigma^2=5$ has been excluded.

duced within the fairly large error bars with a spin cut-off value $\sigma_{\rm res}^2=5$. The errors are given by the finite energy range of data available and are calculated after expressions derived by Hall¹⁵, Böhning ¹⁶, and Gibbs ¹⁷. Within these f.r.d.-errors it was found for all transitions into the various final states of ²³Na that the calculated shape of the cross section agrees with the measured distribution for $\sigma_{\rm res}^2$ lying between 2.5 and 7. The absolute value of the cross section is determined by the ratio of $\Gamma_{1/2}/D_{1/2}$. In Table 2 these ratios corresponding to the possible values of $\sigma_{\rm res}^2$ are listed.

For the $^{37}\text{Cl}(p,\alpha)^{34}\text{S}$ reaction, the experimental averaged cross sections are similar in character to those for magnesium being anisotropic, but slightly antisymmetric about 90° . From other arguments to be discussed later in this section we have reason to assume the presence of direct reaction contribution to the cross section. Because this adds linearly to the average fluctuating cross section the measured cross sections have been corrected by subtracting the direct contribution. The result is seen in Fig. 11. Crosses

¹⁷ W. R. Gibbs, Phys. Rev. 139, B 1185 [1965].

^{14a} Calculations made by G. Marcazzan et al. (private communication) and by Eberhard et al. ¹⁰ show indeed that $\Gamma_{\rm n}/\Gamma_{\rm p} \cong 1$ for the nuclei discussed here.

¹⁶ M. Böhning, C. R. Congr. Intern. Phys. Nucl. Ed. Centre National de la Recherche Scientifique, Paris 1964, Vol. II, p. 697 and to be published.

represent the uncorrected cross sections and the full dots correspond to such experimental values which have been corrected or where no correction has been necessary. The uncertainty connected with the points corrected on direct interaction is not yet clear and simply the f.r.d. errors on the pure statistical cross sections have been drawn. The best Hauser-Fesh-Bach fit to the shape of the angular distribution using a spin cut-off parameter $\sigma_{\rm res}^2=10$ is also seen in Fig. 11. From inspecting the angular distributions for the first and second excited state transitions quoted in an earlier paper ⁵, the allowable value of $\sigma_{\rm res}^2$ lies between 7.5 and 15. From this an upper and lower limit of Γ_0/D_0 were found to be 34 and 20, respectively. (See also Table 2.)

In all three reactions, for the Hauser-Feshbach calculations the proton transmission coefficients have been taken from Meldner and Lindner ¹⁸ and the α-particle penetrabilities from Huizenga and Igo ¹⁹.

We now turn to the discussion of the averaged angular distributions from the reaction 45 Sc (p, α) 42 Ca shown in Fig. 12. The most striking feature is the nearly complete flatness of all measured angular distributions for the transitions into the different final states of ⁴²Ca. As this experiment has certainly vielded even a larger fraction of direct reaction contribution than the former experiment ${}^{37}\text{Cl}(p,\alpha){}^{34}\text{S}$, we tried to fit the data in the same manner as it was described in the Cl-case. A best value for the spin cut-off parameter $\sigma_{\rm res}^2$ of 10 was obtained by performing a fit with a HAUSER-FESHBACH calculation. It must be stressed that the data show evidence for values of σ_{res}^2 between 7.5 and 15 within the error bars, and Γ_0/D_0 lies between 50 and 135. Here the question arises where the large direct reaction contribution to the cross section comes from. As Bayman has pointed out in the Brookhaven Conference 20. Sherr et al. 21 have measured angular distributions in the reaction ${}^{45}\text{Sc}(p, \alpha)$ at 17.8 MeV proton energy. These angular distributions are very similar to those measured at about 9 MeV, being nearly flat and showing no dominant structures. The relative amount of direct reaction contribution in the ground state and in the excited states are equal in our experiment, furthermore the cross section is increasing with increasing spin of the final state which comes out of the Hauser-Feshbach calculations, too. If the ⁴⁵Sc(p, α) cross section was due to a triton pick up mechanism, the cross section should decrease with increasing excitation energy of the final nucleus 42Ca because of the Q-value and of the increasing complexity of the excited states, as Sherr's experiment and Bayman's calculation have shown.

It may be concluded that the estimated ratios of Γ/D summarised in Table 2 are large enough to analyse the observed cross section fluctuations in terms of Ericson's theory.

The most extensive results have been obtained by applying correlation functions to the data. From the Lorentzian of the autocorrelation function the value of the coherence width Γ is extracted. The Γ -values found by this method and by the method of the harmonic analysis of fluctuations 22 are listed in Table 3. This latter method uses the fact that the fluctuating cross section can be regarded as a stationary process. To the excitation function corresponds then a Fourier transformed time dependent process, which has an exponential intensity spectrum. The system decays with a constant being proportional to the coherence energy Γ . This harmonic analysis has been used up til now only in the 45 Sc-case. An

	$\sigma_{ ext{res}}^2$	$\Gamma_{1/2}/D_{1/2}$	Γ_0/D_0	$E_{ m exc}({ m MeV})$
$^{26}{ m Mg}({ m p,}\alpha)^{23}{ m Na}$	$5 - 2.5 \\ + 2.0$	$ \begin{array}{r} 33 + 117 \\ - 9 \end{array} $		20
$^{37}\mathrm{Cl}(\mathrm{p},\alpha)^{34}\mathrm{S}$	$10\ -2.5\ +5.0$	-	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	21.5
$^{45}\mathrm{Sc}(\mathrm{p},\alpha)^{42}\mathrm{Ca}$	$10 - 2.5 \\ + 5.0$	-	$100 + 35 \\ -50$	19

Table 2. The values Γ/D for the different σ_{res}^2 from the Hauser-Feshbach calculations at the corresponding excitation energies of the compound nucleus.

¹⁸ H. Meldner and A. Lindner, Z. Phys. 180, 362 [1964].

¹⁹ J. R. Huizenga and G. Igo, Nucl. Phys. 29, 462 [1962].

²⁰ B. F. BAYMAN, Proc. Summer Study Group on the Physics of the Emperor Tandem Van de Graaff Region, BNL 948 (C-46), 1965, Volume I, p. 67.

²¹ B. F. BAYMAN, H. FUNSTEN, N. R. ROBERSON, E. ROST, and R. SHERR, unpublished.

²² M. Böhning, Max-Planck-Institut für Kernphysik Jahresbericht, Heidelberg 1965.

$\Theta_{ m cm}$	$\Gamma_{ m auto}^{26}({ m p,}lpha_0)^{23}{ m Na}$	$\Gamma_{ m auto}^{26}({ m g}({ m p},lpha_1)^{23}{ m Na}$	$\Gamma_{ m auto}^{26}(m Mg(p,lpha_2)^{23}Na \ \Gamma_{ m auto}(m keV)$	$\Gamma_{ m auto}({ m keV})^{23}{ m Na}$	
176° 171°	56 56	44 48		_ 56	
163°	58	56	_	_	
159°	56	56	64	52	
146°	54	44		_	
133.5°	48	40	52	44	n=26
115°	$\overline{52}$	80			
102°	56	64	_		
82°	54	52	_		
47.5°	52	56	_	_	
33.5°	50	48	_	_	
	$^{37}\mathrm{Cl}(\mathrm{p},\alpha_0)^{34}\mathrm{S}$	$^{37}\mathrm{Cl}(p,\alpha_1)^{34}\mathrm{S}$	$^{37}{ m Cl}({ m p,}\alpha_2)^{34}{ m S}$		
$\Theta_{ m cm}$	$\Gamma_{ m auto}({ m keV})$	$\Gamma_{ m auto}({ m keV})$	$\Gamma_{ m auto}({ m keV})$		
175°	18	21	16		
171°	18	20	16		
163°	19	19	17		
158°	18	20	16		
145°	18	24	17		
132°	19		16		
113°	15		16		
100°	15	15	_		n = 17
80°	16	16			
67°	18	16	22		
46°	14	18			
33°	16	18	16		
	$^{45}\mathrm{Sc}(\mathrm{p},\alpha_0)^{42}\mathrm{Ca}$	$^{45}\mathrm{Se}(\mathrm{p},\alpha_1)^{42}\mathrm{Ca}$	$^{45}\mathrm{Se}\left(\mathrm{p},\alpha_{2}\right)^{42}\mathrm{Ca}$	$^{45}\mathrm{Sc}\left(\mathrm{p},\alpha_{3}\right)^{42}\mathrm{Ca}$	$^{45}{ m Sc}({ m p},\!{ m a_4}){}^{42}{ m Ce}$
$\Theta_{ m cm}$	$\Gamma_{ m auto} \Gamma_{ m frequ} \qquad \Gamma_{ m auto} \Gamma_{ m frequ} \ ({ m keV})$		$\Gamma_{ m auto} \Gamma_{ m frequ} \ m (keV)$	$\Gamma_{ m auto} \Gamma_{ m frequ} \ m (keV)$	$\Gamma_{ m auto} \Gamma_{ m freq} \ m (keV)$
173.3°	8.3 6.6	6.6 6.2	13.3 9.5	8.8 —	7.1 —
164.6°	8.7 7.0	6.5 6.4	12.5 9.5	8.4 —	7.3
156.0°	8.5 7.1	6.6 6.2	12.1 10.0	9.6 —	8.4
138.7°	9.0 7.8	7.6 7.4	9.5 7.9	8.0	6.6
121.2°	8.4 9.1	10.5 8.4	12.1 8.5	9.1 —	6.7
103.4	8.2 6.4	11.6 7.4	11.8 6.8	9.3	5.1 —
85.5°	7.4 —	10.1	11.0	10.0	7.0
63.2°	6.1 —		12.7		7.8
					n=35

Table 3. The mean level width Γ for different angles and different groups in the three reactions studied. The sample size parameter n is also given.

example of this type of analysis is given in Fig. 13. The full line is a least squares fit of an exponential function onto the points of the spectrum. The finite range of data error on Γ is in this case smaller than the corresponding error derived directly from the autocorrelation function. The latter f.r.d.-error ¹⁶ of Γ was found to be about 20% in the cases of ²⁶Mg and ³⁷Cl, and about 17% in the ⁴⁵Sc reaction. Table 3 also contains the sample size parameters n, defined as the measured energy interval divided by π Γ .

As it was pointed out in previous papers $^{4,\,23}$, in the case of the $^{26}{\rm Mg}\,({\rm p},\,\alpha)^{\,23}{\rm Na}$ reaction the results of ordinary correlation function analysis failed to agree with the statistical predictions even when the large f.r.d.-errors were taken into account. In particular it is seen from Fig. 14 which represents the autocorrelation function for the reaction $^{26}{\rm Mg}\,({\rm p},\alpha)$ at $\Theta_{\rm cm}=102.4^{\circ}$ that the calculated function is shifted upwards showing a marked shoulder. The autocorrelation function displaced above the ε -axis

²³ B. W. ALLARDYCE, P. J. DALLIMORE, I. HALL, N. W. TANNER, A. RICHTER, P. VON BRENTANO, and T. MAYER-KUCKUK, Phys. Letters 18, 140 [1965].

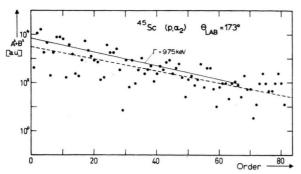


Fig. 13. Frequency spectrum calculated for the excitation function from the $^{45}\mathrm{Se}\left(\mathrm{p},\alpha_{2}\right)$ $^{42}\mathrm{Ca}$ reaction at $\Theta_{\mathrm{lab}}{=}173^{\circ}$. The full line is a result from a weighted least squares fit of an exponential function on to the points of the spectrum.

is oscillating now about a new baseline, so that the width Γ obtained from the half height of $C(\varepsilon)$ varies between 52 keV and 440 keV. The lack of agreement between the experiment and the statistical theory has been removed by a "modulated fluctuation" analysis ^{4, 23} which considers non statistical variations of the mean. The coherence width Γ becomes constant and is of the order of 50 keV. These "demodulated" values of Γ are the ones displayed in Table 3.

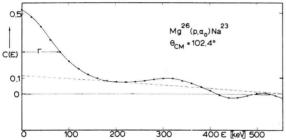


Fig. 14. Autocorrelation function for the reaction 26 Mg (p, α_0) 23 Na at $\Theta_{\rm cm} = 102.4^{\circ}$ calculated from the Heidelberg data only, showing the baseline shift due to modulation.

In the reaction ${}^{37}\mathrm{Cl}(p,\alpha)$ the coherence energy Γ derived from the excitation functions is rather constant with angle and α -group with a mean value of $18~\mathrm{keV}$.

By inspecting the Γ -values from the $^{45}\mathrm{Sc}(\mathbf{p},\alpha)$ reaction it is seen that here the coherence energy varies with particle group more than is compatible with the corresponding f.r.d.-errors. Because Γ in fact is a function of the angular momentum J of the compound states excited, it should be denoted by Γ_J . Whether or not the J-dependence counts for this observed discrepancy can be decided by discussing the measured mean level width in terms of Eq. (1).

For the exact calculation of the *J*-dependence after this formula not only the ratio between the spin cutoff parameter of the compound nucleus and of the residual nuclei $\sigma_c^2/\sigma_{res}^2$ must be known but also the absolute values of each of them are required. From the Hauser-Feshbach approach to the average differential cross section the parameter σ_{res}^2 has been determined within the f.r.d.-errors. The compound nucleus is normally excited about 10 MeV higher than the residual nucleus, so that the spin cut-off parameter σ_c^2 should be larger than σ_{res}^2 . If we assume a rigid body moment of inertia τ_{rig} for the compound nucleus excited at about 20 MeV, and a nuclear temperature T which is proportional to the square root of the excitation energy, σ_c^2 can not exceed the limits given by

$$\sigma_{ ext{res}}^2 \leq \sigma_{ ext{c}}^2 \leq rac{ au_{ ext{rig}} \cdot T}{\hbar^2}$$

For the 45 Sc-case, σ_c^2 is thus restricted to values ≤ 13 , which have to be compared with the best $\sigma_{\rm res}^2 = 10$ from the Hauser-Feshbach fit. Using these values in Eq. (1), $\Gamma_{J=0}/\Gamma_{J=5} = 1.47$ is found. For a comparison of the experimental values of the mean level width Γ with statistical model calculations of Γ_J , the distribution of the partial cross sections $\langle \sigma_J \rangle$ with respect to angular momentum J has to be known and was calculated according to Hauser and Feshbach. For proton energies of about 9 MeV, these distributions exhibit a maximum J-value ranging from 3 to 5 for different angles and final states. These large values of J occur because the target spin is $7/2^-$. By weighting the theoretical values 10 of Γ_J by the relative strength of the $\langle \sigma_J \rangle$ we get

$$I_{
m eff}^{
m th} = rac{\sum_{J} \langle \sigma_{J} \rangle \, I_{J} /}{\sum_{J} \langle \sigma_{J}
angle}.$$
 (3)

Because the location of the maxima in the $\langle \sigma_J \rangle$ -distributions does not change very much with angle and α -particle group, the value of $\Gamma^{\rm th}_{\rm eff}$ is about the same for all angles and final states and therefore the J-dependence of Γ can not explain the variations within the experimentally found Γ -values for different states.

The same attempt was made to investigate the J-dependence of the mean level width Γ in the reaction $^{37}\text{Cl}(p,\alpha)$. Here the distributions of the partial cross sections show different maxima for different angles. This phenomenon is demonstrated in Fig. 15 for the α_1 -transition. From the experiment a constant value of Γ was deduced and the calculated ratio

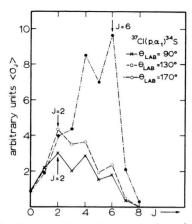


Fig. 15. Distributions of partial cross sections $\langle \sigma_J \rangle$ for the $^{37}\mathrm{Cl}\,(\mathrm{p},\alpha_1)^{34}\mathrm{S}$ reaction, calculated after Hauser and Feshbach for different scattering angles. The maximum J-values are indicated.

 $\Gamma_{\rm eff}^{\rm th}(\Theta)/\Gamma_{\rm eff}^{\rm th}(\Theta')$ confirms this result within the extreme cases when we put the spin cut-off parameter $\sigma_{\rm c}=\sigma_{\rm res}^2$ and $\sigma_{\rm c}=\tau_{\rm rig}\cdot T/\hbar^2$.

The Γ -data of $^{26}{\rm Mg\,}({\rm p},\alpha)^{23}{\rm Na}$ reflect essentially the same behaviour as in the $^{37}{\rm Cl}$ -case. After removing the modulation effect, neither a pronounced experimental variation of Γ has been found nor do the statistical model calculations give any support for a strong J-dependence. In Fig. 16 the partial cross sections for the α_0 -transition as a function of J under different angles are plotted. Here the J-values are half integer because the compound nucleus contains an odd number of nucleons.

Finally, the other quantities which apart from the mean level width Γ come out of the analysis by

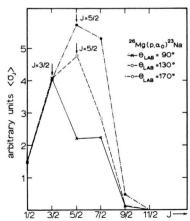


Fig. 16. The same as in Fig. 15 but for the $^{26}{
m Mg}\,({
m p},\alpha_0)^{23}{
m Na}$ reaction.

²⁴ R. O. Stephen, Clarendon Lab. Report, Oxford 1963.

applying correlation functions are found to be also in agreement with the statistical theory. The normalised variance $C_{\varepsilon=0}(\Theta)$ which measures the strength of fluctuations and the cross correlation coefficients which should be zero in purely statistical reactions have been calculated and compared with predictions from the Hauser-Feshbach calculations. The removal of modulation effects in the ²⁶Mg reaction led to a consistency between the theoretically and experimentally obtained values within the f.r.d.-errors 4. In the 37Cl case, however, this consistency was found to be even better, although the direct interaction contribution to the cross section has complicated the interpretation 5. This direct contribution shows up mostly in the 45Sc reaction. From Fig. 17 it can be seen that the experimental

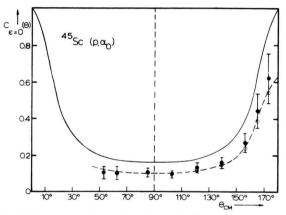


Fig. 17. The comparison of the theoretically predicted and of the experimentally found normalized variances for the reaction $^{45}\mathrm{Sc}(\mathrm{p},\alpha_0)^{42}\mathrm{Ca}$. The experimental values are shifted downwards by the amount of direct interaction. The open circles were derived from the harmonic analysis.

 $C_{\varepsilon=0}(\Theta)$ is shifted downwards by the amount of direct interaction. In the example plotted this amount has a magnitude of about 60% even at 180° where the number of effective channels

$$N_{\rm eff} = 1/C_{\varepsilon=0}(\Theta)$$

is equal to one. The amount of direct contribution has been evaluated by the help of Stephen's formula 24 . To avoid complications entering the analysis due to the experimental energy resolution which is about half the mean level width Γ in the 45 Sc case and which would reduce the normalised variance in the same manner as the direct interaction contribution 25 , the experimental normalised variances have been corrected on this fact. Also the decreasing

²⁵ H. L. Acker, Tandem Lab. Report, Heidelberg 1964.

average illustrated in Fig. 9 has been taken into account as a linear and additive energy dependent effect. The cross correlation coefficients with respect to the transitions into the different final states are small in all three reactions, indicating the statistical character of the observed fluctuations.

Conclusion

In the previous sections a statistical model analysis on the three reactions 26 Mg (p, α) 23 Na, 37 Cl (p, α) 34 S, and 45 Sc (p, α) 42 Ca has been presented. By analysing the averaged angular distributions using the Hauser and Feshbach formalism and by comparing the values deduced from the data by means of correlation functions with the theoretical predictions the statistical model has been found to be successful. As it was pointed out by other autors 26 , too, who measured excitation function in the region $A \cong 25$ the statistical compound cross section is possibly distorted by non statistical processes as it shows up in the

 26 Mg (p, α) 23 Na reaction, but no significant direct interaction contribution to the cross section has been found here. The amount of direct contribution, however, observed in the 37 Cl-reaction even increases to the heavier nucleus 45 Sc. The data also provide an information about the dependence of the mean level width Γ in the compound nucleus on angular momentum J. In all three reactions this J-dependence has been found to be weak.

We wish to thank Professor W. Gentner for this generous support given to this work.

Note added in proof: W. R. Gibbs recently has pointed out to us that the physical quantity beeing averaged should not be the level width but rather the lifetime of the compound nucleus. This would change formula (3) into the expression

$$(\Gamma_{\rm eff}^{\rm th})^{-1} = \sum_{J} \frac{\langle \sigma_J \rangle}{\sum_{I} \langle \sigma_J \rangle \Gamma_J}. \tag{3 a}$$

If we use eq. (3 a) instead of eq. (3), our conclusions given above remain unaffected although the ratio of

$$\Gamma_{\rm eff}^{\rm th} (\Theta) / \Gamma_{\rm eff}^{\rm th} (\Theta')$$

becomes slightly more sensitive on the ratio of $\sigma_c^2/\sigma_{res}^2$.

655 [1965]. — R. V. Elliott and R. H. Spear, Australian National University, Canberra, Department of Nuclear Physics Annual Report [1965].

²⁶ P. P. Singh, R. E. Segal, L. Meyer-Schützmeister, S. S. Hanna, and R. G. Allas, Nucl. Phys. 65, 577 [1965]. — I. F. Bubb, J. M. Poate, and R. H. Spear, Nucl. Phys. 65,